[a] Using iteration, guess an explicit formula for the sequence (ie. a general formula for a_n).

$$a_1 = 5a_0 + 8 = \lfloor 5 \cdot 2 + 8 \rfloor$$

$$a_2 = 5a_1 + 8 = 5 \cdot (5 \cdot 2 + 8) + 8 = 5^2 \cdot 2 + 5 \cdot 8 + 8$$

$$a_3 = 5a_2 + 8 = 5 \cdot (5^2 \cdot 2 + 5 \cdot 8 + 8) + 8 = 5^3 \cdot 2 + 5^2 \cdot 8 + 5 \cdot 8 + 8$$

$$a_n = 5^n \cdot 2 + (5^{n-1} \cdot 8 + \dots + 5 \cdot 8 + 8) = 2 \cdot 5^n + \frac{8(5^n - 1)}{5 - 1} = 4 \cdot 5^n - 2$$

[b] <u>Using mathematical induction</u>, prove that your formula in [a] is correct.

Proof by Mathematical Induction:

Basis step:
$$|a_0| = 4 \cdot 5^0 - 2 = 4 - 2 = 2$$

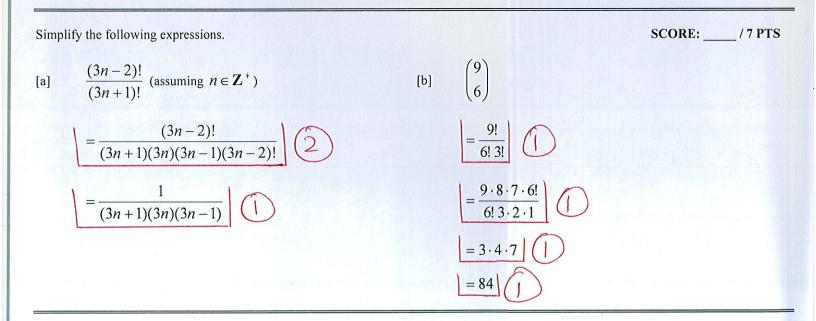
Inductive step: Assume that $a_k = 4 \cdot 5^k - 2$ for some particular but arbitrary integer $k \ge 0$

[Prove that
$$a_{k+1} = 4 \cdot 5^{k+1} - 2$$
]

$$k \ge 0$$
, so $k+1 \ge 1$

So,
$$a_{k+1} = 5a_k + 8$$
 = $5(4 \cdot 5^k - 2) + 8$ = $4 \cdot 5^{k+1} - 10 + 8$ = $4 \cdot 5^{k+1} - 2$

By mathematical induction $a_n = 4 \cdot 5^n - 2$ for all non-negative integers n



One of the following statements is true and one is false.

/13 PTS SCORE: State clearly which statement is false, show that it is false, then write a formal proof for the true statement using mathematical induction.

- $(n^3 + 2n 1) \mod 3 = 2$ for all positive integers n
- $3 | (n^3 n^2)$ for all non-negative integers n

[a] is true.

Proof by Mathematical Induction:

Basis step:
$$(1^3 + 2 \cdot 1 - 1) \mod 3 = 2$$

Basis step:
$$(1^3 + 2 \cdot 1 - 1) \mod 3 = 2$$
 since $2 = 3 \cdot 0 + 2$

Inductive step: Assume that
$$(k^3 + 2k - 1) \mod 3 = 2$$
 for some particular but arbitrary $k \in \mathbb{Z}^+$

[Prove that
$$((k+1)^3 + 2(k+1) - 1) \mod 3 = 2$$
]

$$k^3 + 2k - 1 = 3q + 2$$
 for some $q \in \mathbb{Z}$ by definition of mod

$$k^3 + 2k - 1 = 3q + 2$$
 for some $q \in \mathbb{Z}$ by definition of mod
$$(k+1)^3 + 2(k+1) - 1 = k^3 + 3k^2 + 3k + 1 + 2k + 2 - 1 = k^3 + 2k - 1 + 3(k^2 + k + 1)$$

So,
$$((k+1)^3 + 2(k+1) - 1) \mod 3 = 2$$
 by definition of mod

By mathematical induction
$$(n^3 + 2n - 1) \mod 3 = 2$$
 for all positive integers n

[b] is false. If n=2, $n^3-n^2=4$ and $3 \nmid 4$. (2) MUST HAVE VALID COUNTEREXAMBLE